Goal-directed state equation for tracking reaching movements using neural signals

Lakshminarayan Srinivasan[*,†,‡], Uri T. Eden[†,‡], Alan S. Willsky[*], and Emery N. Brown[†,‡]
[*]Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology
[†]Division of Health Sciences & Technology, Harvard / MIT
[‡]Department of Anesthesia and Critical Care, Massachusetts General Hospital

***Abstract*— This paper addresses the problem of estimating reaching movements. We derive a Bayesian-optimal discrete-time state equation to support real-time filters that incorporate observations about the target position and arm trajectory. The resulting algorithm is compatible with any filtering method, such as point process or Kalman filters, and any recording modality, such as multielectrode arrays, intracortical EEG, or eye trackers.**

***Index Terms*— neural prosthetics, reaching arm movements, estimation, decoding, state evolution equation, stochastic control**

I. Introduction

MANY models have been used to describe the motion of free or reaching human arm movement. Reaching models are often described in the control framework as state equations with a forcing term optimized for some cost criterion like energy or target error [1]. A statistical modeling approach based on an empirical database of movements was recently described [2]. In contrast, the free arm movement estimation literature has largely relied on either no models or random-walk-type models, both for computational simplicity and for robustness in the face of uncertain movement constraints [3].

Our objective is to develop a method that optimally combines this generic model of free arm movement with information about the target state to produce a generic model for reaching arm movement. Uncorrelated increments are also desired to facilitate use with real-time recursive estimation procedures such as Kalman filters.

The model represents a set of priors for any method of estimating arm movements, including point process filters, Kalman filter variants, particle filters, or general probabilistic inference. Measurements from any device or brain region can be incorporated into this estimation procedure, including local field potentials and spiking activity.

This paper adapts continuous time surveillance methods [4] for discrete time. The derivation presented here follows an approach similar to a discrete-time backwards Markov model construction [5].

In Section II, we present the goal-directed reach state equation. In Section III, we describe an augmented state space to accommodate concurrent target dynamics. In Section IV, we apply these methods to estimate arm movement from simulated cortical activity during a reach.

This work was supported in part by the NIH Medical Scientist Training Program Fellowship to LS and NIH grant R01 DA015644 to ENB. Correspondence can be addressed to ls2@neurostat.mgh.harvard.edu

II. Incorporating target information

We begin with any P-dimensional linear time-varying discrete-time state space model of free arm movement. At time t, the vector x_t describes the arm state, A_t is an invertible state transition matrix, and w_t is a zero mean Gaussian increment:

$$x_t = A_t x_{t-1} + w_t \tag{1}$$
$$E[w_t w'_\tau] = Q_t \delta_{t-\tau} \tag{2}$$

The initial condition is specified as

$$x_0 \sim N(m_0, \Pi_0) \tag{3}$$

Consider an observation of target position specified with uncertainty for known arrival time T:

$$y_T = x_T + v_T \tag{4}$$

where $v \sim N(0, \Pi_T)$ denotes the observation noise. We now seek the Bayes least squares estimate of each increment w_t conditional on the noisy observation y_T of the target location and the present location x_{t-1}. This is equivalent to the linear least squares estimate (LLSE) for jointly Gaussian distributions. The standard LLSE estimate of w_t from y_T is given by

$$\hat{w}_t(y_T) = E[w_t] + cov(w_t, y_T)cov^{-1}(y_T, y_T)(y_T - E[y_T]) \tag{5}$$

The new increment corresponds to the error of the LLSE estimate, with covariance given by the formula

$$cov(\epsilon_t) = cov(w_t) - cov(w_t, y_T)cov^{-1}(y_T)cov'(w_t, y_T) \tag{6}$$

The expected value of w_t is zero. To derive expressions for the expected value and covariances involving y_T, we first write y_T in terms of x_{t-1}, v_T, and the intervening increments w_i for $t \le i \le T$; that is

$$y_T = \phi(T, t-1)x_{t-1} + \sum_{i=t}^{T} \phi(T, i) w_i + v_T \tag{7}$$

where $\phi(t, s)$ denotes the state transition matrix that maps the state x_s to the state x_t,

$$\phi(t, s) = \prod_{i=s+1}^{t} A_i^{sign(t-s)} \tag{8}$$

Calculate the mean and covariances of y_T and substitute into the LLSE estimation formula (5) to obtain the mean of the increment based on target information,

$$\begin{aligned}\hat{w}_t(y_T) &= Q_t\phi'(T,t) \\ &\times[\Pi_T + \sum_{i=t}^{T}\phi(T,i)Q_i\phi^T(T,i)]^{-1} \\ &\times[y_T - \phi(T,t-1)x_{t-1}] \qquad (9)\end{aligned}$$

where

$$\Pi_T = E[v_T v_T] \qquad (10)$$

Calculate the error covariance from (6) and simplify to obtain

$$cov(\epsilon_t) = Q_t - Q_t\Pi^{-1}(t,T)Q_t' \qquad (11)$$

where

$$\Pi(t,T) = \phi(t,T)\Pi_T\phi'(t,T) + \sum_{i=t}^{T}\phi(t,i)Q_i\phi'(t,i) \qquad (12)$$

Hence, the increment w_t given a known x_{t-1} and noisy target observation y_T has mean $\hat{w}_t$ and covariance $cov(\epsilon_t)$ as calculated above.

In practice, the quantity $\Pi(t,T)$ is computed recursively starting at $\Pi(T,T)$. To derive this result, compare the equations for $\Pi(t-1,T)$ and $\Pi(t,T)$. The consequent recursion is

$$\begin{aligned}\Pi(t-1,T) &= \phi(t-1,t)\Pi(t,T)\phi'(t-1,t) \\ &\quad+\phi(t-1,t)Q_t\phi'(t-1,t) \qquad (13) \\ \Pi(T,T) &= \Pi_T + Q_T \qquad (14)\end{aligned}$$

For estimation, $\Pi(t,T)$ can be recomputed with each new observation, intermittently, or not at all.

To complete the equivalent reach state equation, the initial state and covariance are updated with standard LLSE formulas,

$$\begin{aligned}\Pi_s &= (\Pi_0^{-1} + \Pi^{-1}(0,T))^{-1} \qquad (15) \\ x_s &= \Pi_s(0)(\Pi_0^{-1}m_0 + \Pi^{-1}(0,T)\phi(0,T)y_T) \qquad (16)\end{aligned}$$

In summary, the equivalent state equation for reaching movements is given by

$$\begin{aligned}x_t &= A_t x_{t-1} + u_t + \epsilon_t \qquad (17) \\ u_t &= \hat{w}_t(y_T) \qquad (18) \\ \epsilon_t &\sim N(0, cov(\epsilon_t)) \qquad (19) \\ x_0 &\sim N(x_s, \Pi_s) \qquad (20)\end{aligned}$$

The process resulting from the new state equation (17) is also Markov, because x_t is a function of the state at only the preceding timestep x_{t-1}. Additionally, based on the orthogonality principle, the ϵ_t are uncorrelated. A proof will be included in subsequent correspondence.

The above form (17) provides some intuition about the evolution of the arm trajectory under this reaching model. As the hand moves closer to the target in time, the trajectory becomes more constrained by the target location. This is accomplished by a combination of the forcing term u_t and the noise term ϵ_t. With time, the forcing term more insistently pushes the arm to a path that sets it on course for the target. The covariance in the noise term tapers in proportion to target uncertainty as it becomes more apparent that specific changes in state are needed to bring the arm to the target at time T.

III. Updating Target Estimate

Observations about the reach path can also be used to refine estimates of the target if path and target are dependent. To support recursive estimation of target position, we augment the state space with x_T to include target position and target velocity variables. The resulting state equation is written as

$$\begin{bmatrix} x \\ x_T \end{bmatrix}_t = \begin{bmatrix} \Psi & \Gamma \\ 0 & I \end{bmatrix} \begin{bmatrix} x \\ x_T \end{bmatrix}_{t-1} + E_t \qquad (21)$$

where

$$\begin{aligned}\Psi &= A_t - Q_t\phi'(t-1,t)\Pi^{-1}(t-1,T) \qquad (22) \\ \Gamma &= Q_t\phi'(t-1,t)\Pi^{-1}(t-1,T)\phi(t-1,T) \qquad (23)\end{aligned}$$

and the increment is

$$E_t = \begin{bmatrix} \epsilon_t & 0 \end{bmatrix}' \qquad (24)$$

Uncertainty can be added to this increment to track possible drifts in target states.

This one state equation now supports real-time decoding of concurrent or sequential neural activity relating in various degrees to the path and target of the reach. When only trajectory information is available, the model reduces to free-arm movement where u_t is zero and ϵ_t retains the same statistics as w_t. When only target information is available, the model dynamics will travel to the target in proportion to uncertainty about the target based on u_t. Because the target position and velocity are now state variables, refined estimates of the target are available even as the reach proceeds.

IV. Applications

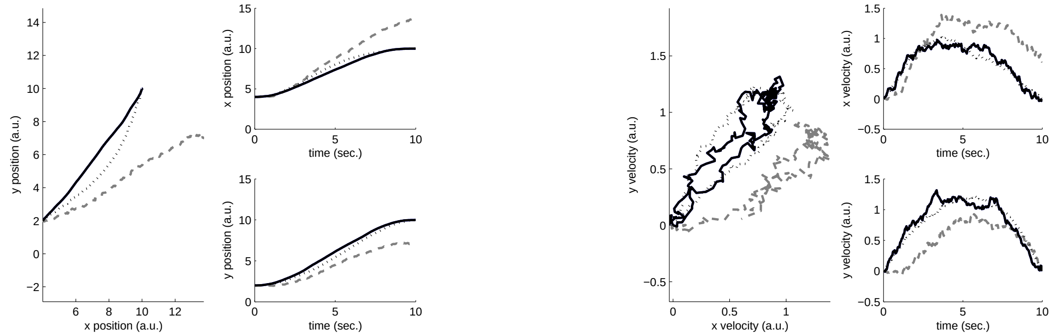
To illustrate the flexibility of the above reach state equation, we simulated neural spiking data from primary motor cortex in response to a simulated two dimensional trajectory. A linear-update point process filter based on the probabilistic principles of Kalman filters was employed to reconstruct the trajectory using a free arm movement state equation and the reach state equation. The point process filtering method has been derived and illustrated in detail [7].

The general point process observation equation for the i^{th} neuron at time t is approximated as

$$\begin{aligned}Pr(\Delta N_t^i|x_t, H_t^i) &= exp[\Delta N_t^i log(\lambda(t|x_t, H_t^i)\delta) \\ &\quad -\lambda(t|x_t, H_t^i)\delta] \qquad (25)\end{aligned}$$

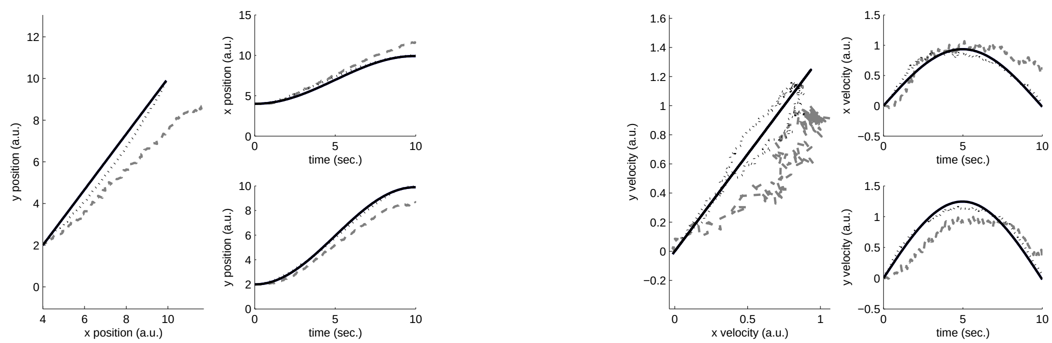
where H_t^i denotes the history of state x_t and spikes ΔN_t^i per time interval δ up to (but excluding) timestep t. The conditional intensity function used here follows primary motor neuron cosine tuning [8], given by

$$\lambda(t|v_x, v_y) = exp(\beta_1 + \beta_2 v_x + \beta_3 v_y) \qquad (26)$$



(a) (b)

Fig. 1. Reconstruction of reaching arm movements from simulated spiking activity. The reach model (17) was first used to generate reach trajectories. Point process filtering reconstructions using a free movement state equation (grey, dashed) and a reach movement state equation (black, dotted) were compared against true movement values (black, solid). (a) Position and (b) velocity reconstructions are plotted for one trial with almost perfectly known target (initial target position and velocity variances of 0.001 a.u.^{2}).



(a) Position (b) Velocity

Fig. 2. Reconstruction in the face of model violation. Now the model used to generate reach trajectories is an appropriately scaled cosine velocity profile. Again, results are compared for point process filtering using free (grey, dashed) and reach (black, dotted) movement state equations against true values (black, solid). Target position and velocity variance are again almost perfectly known to the filter (initial target position and velocity variances of 0.001 a.u.^{2})

Performance in decoding reaching movements using the free arm movement model (1) was first compared against results using the reach state equation (17). Results are illustrated for one trial in Fig. (1). While using the free movement model allows estimation of trajectory, use of the reach model with almost certain knowledge of the target guarantees that the estimated trajectory will converge to the target. Moreover, trajectory reconstructions appear to be improved over the entire duration of the reach.

Decoding under a simple model violation was then examined. In Fig. 2, a canonical scaled cosine velocity model was employed to generate a sample reach trajectory. The movement was then again reconstructed using a point process filter with free and reaching movement models. Again, while both models tracked the reach, the reach movement model tracked more closely by incorporating target information.

The refinement of target estimates from trajectory related neural activity was then illustrated. Because the target state augmented reach model (21) was employed, target estimates were refined with each sample of neural data observed during the movement. In Fig. 4, the uncertainties in target position and velocity estimates are plotted against time samples over the full course of the reach. The rates with which uncertainty drops from initial variances of 0.1 a.u.^{2} depend on the particular distribution of tuning curves in the selected neural ensemble. The target position and velocity uncertainty have characteristic shapes dependent on the form of the state and observation equations.

Finally, the mean squared errors for trajectory reconstruction were examined as the target state became more uncertain. As expected, the errors converged to the free movement error as the initial target position and velocity state variance grew.

The individual performance of the models is specific to the set of neuron tuning curves that is chosen. Additionally, the

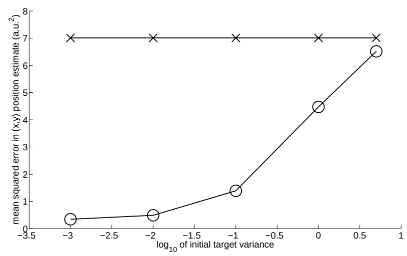


Fig. 3. Mean squared error in position for the reach model (circles) approaches that of the free movement model (crosses) as uncertainty in the target position increases.

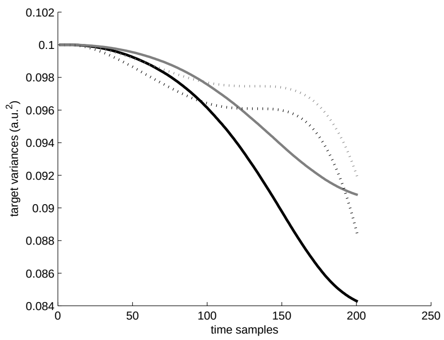


Fig. 4. Target estimate variances over the entire duration of a reach for position (solid) and velocity (dashed), in both x (grey) and y (solid) directions for one trial. Target position and velocity variances are initially 0.1 a.u.^{2}.

relative improvement observed in Fig. 1 and Fig. 2 using the reach model over the free model will also be specific to the quality and quantity of any particular ensemble of neurons. For example, if the ensemble is perfectly informative about the trajectory, no additional benefit is gained from neural-derived observations on the target state.

V. Conclusion

The reach state equation represents the Bayesian-optimal extension of the general autoregressive state equation that has been popular in the decoding of free arm movements with Kalman and point process filters. The resulting model is the minimum constraint placed on a free arm movement based on observation of its target state.

The model is a prior on reach states, and can be used in conjunction with estimation procedures that employ any modality, such as local field potentials or spikes, or any formulation of an observation equation that describes the relationship between the measured signals and desired reaching movement. The result is being extended to accommodate uncertain reach duration.

One appealing feature of the model is that by switching between informative and uninformative target estimates, the reconstruction can alternate between reaching and free arm movements. For neural prosthetics, preserving the dynamics of the system between reaching and free movement modes may prove easier for the user than switching to an entirely different set of priors between these two movement modes.

Because the reach state equation places minimal constraints on the motion, it is expected to be more robust to model violations than trajectory models derived from optimization of specialized cost functions [1], or models based on empirical databases [2]. This may be important in providing the user both flexibility and control. The algorithm also provides a closed form solution and may be more efficient and easy to implement than methods requiring numerical optimization.

The reach state equation is a basic example of a closed loop stochastic control model in that the forcing term is determined by the current and end states rather than a preformed control sequence. Future approaches might examine the use of Monte Carlo methods with nonlinear system dynamics or stochastic optimal control with relatively unconstrained cost functions such as minimum target variance.

Acknowledgment

LS thanks Ali Shoeb, Benjie Limketkai, Ashish Khisti, and Dr. Rengaswamy Srinivasan for discussions.

References

[1] E. Todorov. “Optimality principles in sensorimotor control”, *Nature Neurosci.*, vol. 7, no. 9, pp. 907-915, Sept. 2004.

[2] C. Kemere, G. Santhanam, B.M. Yu, S. Ryu, T. Meng, and K.V. Shenoy. “Model-Based Decoding of Reaching Movements for Prosthetic Systems”, *Proc. 26th Annu. Int. Conf. IEEE EMBS*, San Francisco, CA, USA, pp.4524-4528, Sept. 1-5, 2004.

[3] A.B. Schwartz. “Cortical Neural Prosthetics”, *Annu. Rev. Neurosci.*, vol. 27, pp. 487-507, March 2004.

[4] D.A. Castanon, B.C. Levy, and A.S. Willsky. “Algorithms for the incorporation of predictive information in surveillance theory”, *Int.J.Systems Sci.*, vol. 16, no. 3, pp. 367-382, 1985.

[5] G. Verghese and T. Kailath. “A Further Note on Backwards Markovian Models”, *IEEE Trans. Info. Theory*, vol. IT-25, no. 1, Jan. 1979.

[6] Alan S. Willsky. *6.433 Recursive Estimation: Supplementary Notes*, MIT Course 6.433 Notes, Topic 5.4, p. 18, 1994.

[7] U.T. Eden, L.M. Frank, R. Barbieri, V. Solo, and E.N. Brown, “Dynamic Analysis of Neural Encoding by Point Process Adaptive Filtering”, *Neural Computation*, vol. 16, no. 1, 2004.

[8] D. Moran and A. Schwartz, “Motor cortical representation of speed and direction during reaching,” *J. Neurophys.*, vol. 82, no. 5, pp. 2676-2692, 1999.